Dynamical spatial land-use models

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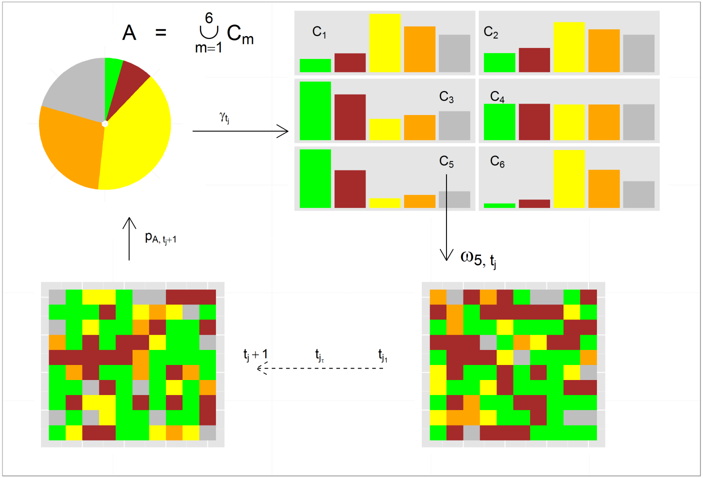
Objective and ideas

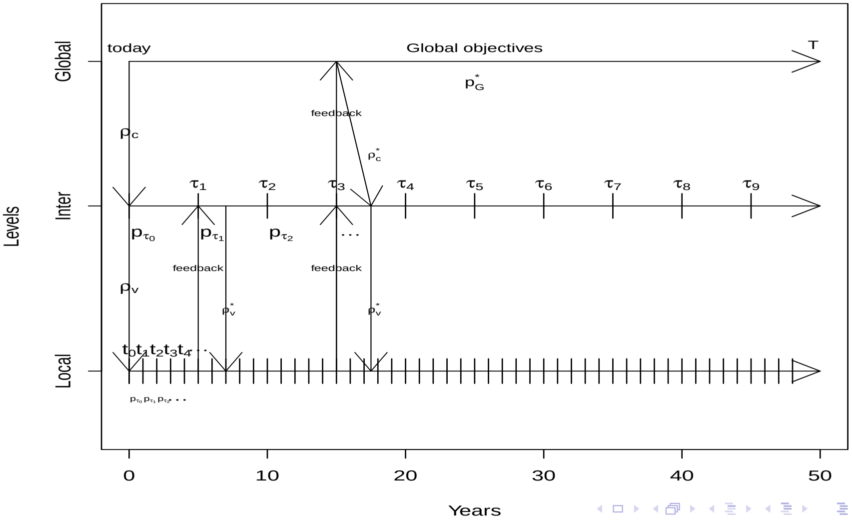
We focus on different spatial nested levels: (i) Congo Basin, (ii) country, (iii) region, (iv) village, ...

Objective:
modeling the evolution of activities in time and space at each level and their interactions subject to global objective constraints.

Objective and ideas: graphical representation



Objective and ideas: graphical representation



Example: One country, three villages

- a country aims to reach:
  30 % crop, 50% of (natural or secondary) forests, and 20 % dedicated to others.
- 3 villages with relative surfaces $\delta_v = c(1/2, 1/3, 1/6)$, with following activities

activities	$\mathbf{p}_{v_1}$	$\mathbf{p}_{v_2}$	$\mathbf{p}_{v_3}$	Observed: $\mathbf{p}_c^0$	target: $\mathbf{p}_c^\star$
Crop	10%	30%	27%	19.5%	30%
Forests	65%	51%	23%	53.3%	50%
Others	25%	19%	50%	27.2%	20%

Question 1:
Find an efficient way to distribute $\mathbf{p}_c^\star$ into the 3 villages

Example:

Solution 1:
Minimal effort

1. Minimal effort

$$\mathbf{p}^\star = (p_1^\star, \ldots, p_V^\star) = \arg\min_{\pi} \sum_{v}^{V} \delta_v \sum_{k}^{K} ||\pi_{kv} - p_{kv}^0||_l$$

   where $||x||_l$ is the L_l-norm (distance).

2. under the following constrains:
   - global constraints

$$\sum_{v=1}^{V} \delta_v \pi_{kv} = p_{kc}$$

   - probability constraints

$$\sum_{k=1}^{K} \pi_{kv} = 1 \quad \text{and} \quad p_{kv} \in [min_{kv}, max_{kv}],$$

     where $min_{kv} \geq 0$ and $max_{kv} \leq 1$ are defined by users.

Objective and gradient functions

Objective function

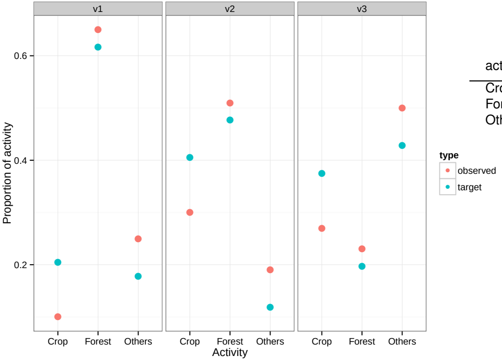
Rewritting the objective function and constraints, the problem can be expressed as:

$$
\begin{aligned}
f(\pi) &= \sum_{v}^{V-1} \delta_v \left[\sum_{k=1}^{K-1} ||\pi_{kv} - p^0_{kv}||_l + ||\sum_{k=1}^{K-1} (\pi_{kv} - p^0_{kv})||_l \right] \\
&+ \delta_V \left[\sum_{k=1}^{K-1} || \frac{1}{\delta_V} \left(p_{kc} - \sum_{v=1}^{V-1} \delta_v \pi_{kv} \right) - p^0_{kV}||_l + || \frac{1}{\delta_V} \left(p_{Kc} - \sum_{v=1}^{V-1} \delta_v \left(1 - \sum_{k=1}^{K-1} \pi_{kv} \right) \right) - p^0_{KV}||_l \right]
\end{aligned}
$$

with $(K-1)(V-1)$ free parameters and $p_{kv} \in [min_{kv}, max_{kv}]$

The gradient functions (first derivatives) as well as R code have been written.

Example: Minimal effort and Conditional chain rule



activities	$\mathbf{p}^0_{v_1}$ ($\mathbf{p}^\star_{v_1}$)	$\mathbf{p}_{v_2}$ ($\mathbf{p}^\star_{v_2}$)	$\mathbf{p}_{v_3}$ ($\mathbf{p}^\star_{v_3}$)
Crop	10% (20.5%)	30% (40.5%)	27% (37.5%)
Forests	65% (61.7 %)	51%(47.7%)	23% (19.7%)
Others	25% (17.8%)	19% (11.8%)	50% (42.8%)

Question 2:
Modeling the observed dynamics of the activities for each village

Example: dynamics in one village $\mathbf{p}_v^0$

Solution 2: Multivariate autoregressive process

Consider the following transformation:

$$\mathbf{p} \quad \mapsto \quad \left(\log\left(\frac{p_1}{p_K}\right), \ldots, \log\left(\frac{p_{K-1}}{p_K}\right)\right) = \xi$$

at the village level, the "observed" dynamics is assumed to be modeled by a vector autoregressive model of order one (VAR(1)).

$$\xi_{kv}^0(t + \Delta_v t) = \mu_{kv} + \sum_{l=1}^{K-1} \beta_{lk} \xi_{lv}^0(t) + \sum_{r}^{R} \theta_v^r x_v^r(t + \Delta_v t) + \varepsilon_{kv}(t + \Delta_v t)$$

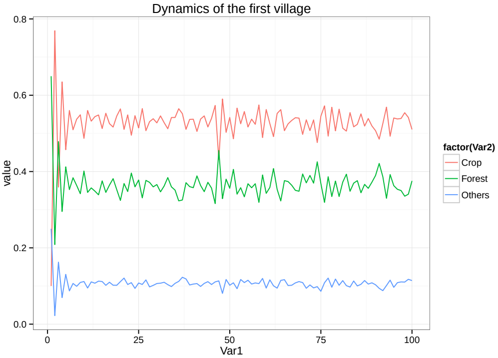
where β and θ are unknown parameters associated to previous activities and environmental covariates, and ε_k is a logistic-Gaussian process such that $\mathbb{C}ov\left(\varepsilon_{kv}(\tau), \varepsilon_{k'v}(\tau')\right) = \sigma_{kk'v}$ if $\tau = \tau'$ and 0 elsewhere.

Example: dynamics in one village $\mathbf{p}_v^0$

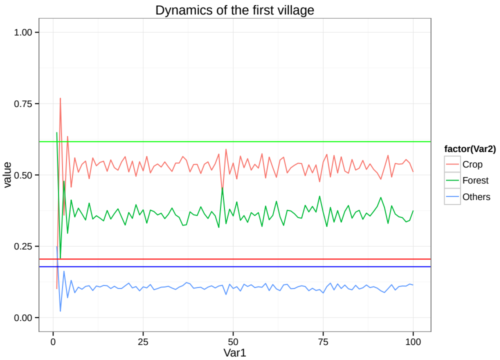
$$\mu = c(1.91, 1.19) \qquad \text{and}$$

β	
-0.86	0.89
-0.46	0.64

activities	$\mathbf{p}_{v_1}$	$\mathbf{p}_{v_2}$	$\mathbf{p}_{v_3}$	target: $\mathbf{p}_c^\star$
Crop	10%	30%	27%	30%
Forests	65%	51%	23%	50%
Others	25%	19%	50%	20%



Example: dynamics in one village $\mathbf{p}_v^0$



natural dynamics do not reach the objectives

Question 3:
Modeling the stimulus process

Construction of stimulus process

Solution 1:
mixture modeling approach

at the village level, objective is to reach $\mathbf{p}_v^\star$
Stimulus processes can act on β parameters.

1. estimate parameters β: $f(\hat{\beta}) \equiv \mathcal{N}(\beta, \Sigma_{beta})$
2. for one observed time, and objectives $\mathbf{p}_v^\star$, calculate

$$\beta^\star(t) \quad \text{such as} \quad \phi(p_{kv}^\star) = \xi_{kv}^\star = \mu + \sum_{l=1}^{K-1} \beta_{lk}^\star(t)\xi_{kv}(t)$$

under constraints that:

$$\xi_v^\star = \mu + \beta^\star \xi_v^\star$$

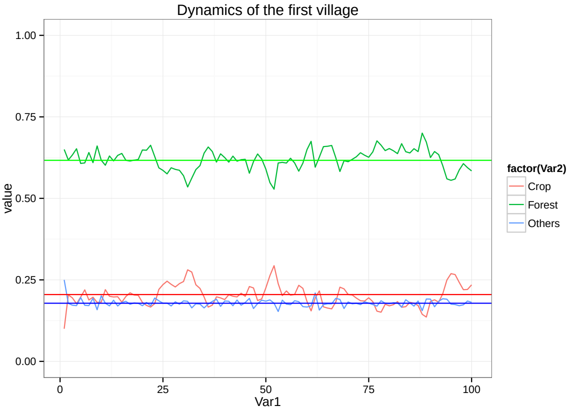
The Stimulus process is defined as a ρ such that:

$$\beta^{Stim} \equiv \rho \times \beta^\star + (1-\rho) \times \hat{\beta}$$

Example: modeling stimulus process

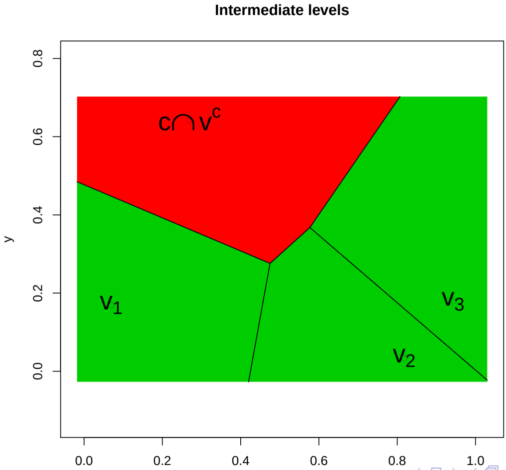
If $\rho = 1$ and $\mu = c(1.91, 1.19)$ and

$\beta^\star$	
-0.56	0.15
-0.87	0.24



Multiscale generalization: country with its own dynamics

Each country is assumed to be divided in two areas:



dynamics on activities at the country level

$$p^0_{kc}(\tau + \Delta_c\tau) \quad = \quad \left(\delta_{c\cap v}\sum_{v=1}^{V}\delta_v p^0_{kv}(\tau + \Delta_c\tau) + \delta_{c\setminus v}p^0_{kc\setminus v}(\tau)\right) \oplus \varepsilon_c(\tau + \Delta_c\tau)$$

and

$$\xi^0_{k\setminus v}(\tau) = \mu_{kc\setminus v} + \sum_{l=1}^{K-1}\omega_{lk}\xi^0_{lc\setminus v}(\tau - \Delta_c\tau) + \sum_{r}^{R}\theta^r_{c\setminus v}x^r_v(\tau) + \zeta_{kc}(\tau)$$

novelty at the country level:

1. $\mathbf{p}_c$ can deal with more activities, such as mines, ...
2. country able to act on relative surface of each village δ_v by increasing or decreasing it.

Questions that need to be answered

If you agree with the dynamics models,

1. Are there any data available to fit the model? In what format are they available? Can they be used now ?
2. Who will help us for the analysis? Calibration, validation...
3. Who will takes care (with us) of the programming/coding? Relevant R packages exist, some simple code are available, but the bulk of the work remains to be done
4. how to calibrate ρ, $\beta^\star$, or $\tilde{\omega}$?